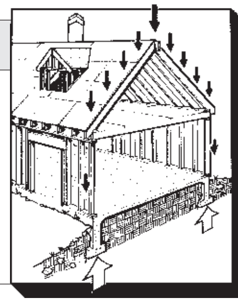
Simple Beam Sizing for Shear and Deflection

by Harris Hyman, P.E.



Last month, we analyzed the floor structure of an 18x20-foot living room. The floor system has an 18-foot-long central girder supporting 10-foot-long 2x8 joists running to the outside walls. The joists are #2 Doug fir 2x8s 16 inches on-center. Using a design floor load of 40 pounds per square foot, we calculated the load on the girder to be 33.3 pounds per inch. Through analysis of bending strength, we arrived at a beam with enough raw strength to handle this load — a quadruple 2x12 girder built up from select Doug fir. But raw strength is only *one* of the elements we have to check out in designing beams. In this column, we'll look at two other ways a beam can fail — *shear* and *deflection*.

Checking for Shear

Shear is a word that is used a little casually. It actually has *two* meanings: One is "to cut off," as when you shear a branch with hedge trimmers. In our case, the relevant meaning is "to slide by in a parallel plane," as when a rock layer in the earth fractures during an earthquake and two strata slide past each other. To see shear in action, try a little experiment. Close this magazine, hold it flat in front of you, then fold it in half vertically. At the unbound edge, the pages slip by one another to form a knife edge — an example of shear. Now fold the magazine again, this time pinching the unbound edge. Now there is a bulge inside the magazine, as the pinch resists the shear.

Shear in wood. Shear is a natural consequence of bending. When a beam is flexed, it goes into a curve. The distance along the outside of the curve — the bottom of the beam — is longer than the distance around the inside — the top of the beam. As the bending stresses force the wood to take on different lengths along its two edges, the wood fibers begin to tear apart as they "slide by" one another parallel to the length of the beam.

Checking for shear is a three-part process: First, we find the *maximum shearing force (Vmax)* that the load will impose on the beam. Next, we use *V* to calculate the *maximum shear stress (fv)* that will develop in the beam as a result of the shearing force. Finally, we check *fv* against *Fv, the maximum allowable shear stress* for the species of wood we're working with — select Doug fir, in the case of our problem beam.

In last month's column, we identified the girder as a "simple beam with a uniformly distributed load." From our reference book, the formula for maximum shearing force (*Vmax*) for this type beam is:

$$V_{max} = w \times \frac{L}{2}$$

$$= 33.3 \text{ lb./in.} \times \frac{(18 \text{ ft.} \times 12 \text{ in./ft.})}{2}$$

$$= 3{,}596 \text{ lb.}$$

where *w* is the unit load on the beam and *L* is its total length.

The maximum shear stress (*fv*) in the beam is calculated by dividing the shearing force (*V*) by the cross-sectional area (*A*) of the beam and multiplying by a factor of 1.5:

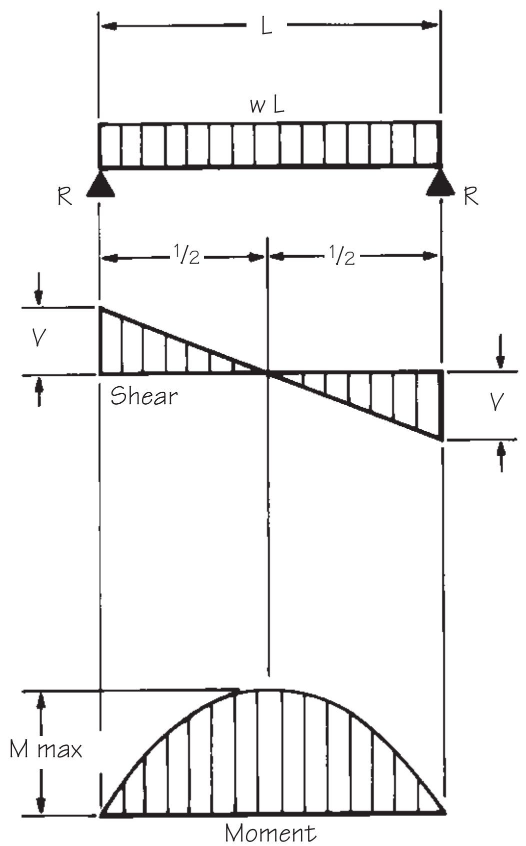
$$fv = 1.5 \times \frac{V}{A}$$

$$= 1.5 \times \frac{3{,}596 \text{ lb.}}{6 \text{ in.} \times 11.25 \text{ in.}}$$

$$= 80 \text{ lb./sq. in.}$$

The *National Design Specification Supplement* (published by American

Simple Beam With Uniform Load



Deflection (D) formula:

$$D_{max} = \frac{5 \times w \times L^4}{384 \times E \times I}$$

Shear formula:

$$V_{max} = \frac{w \times L}{2}$$

Bending moment (M) formula:

$$M_{max} = \frac{w \times L^2}{8}$$

Engineers use schematic drawings like this to model various loading conditions. The top sketch represents the beam's uniform load. The reactions (R) are at the support points. The middle sketch indicates that shear stress increases uniformly from none at center span to the greatest shear at the ends of the beam. The moment sketch, at the bottom, shows that bending forces increase toward the center of the beam, with the maximum bending at center span.

Forest & Paper Assn.; 202/463-2700) gives a maximum allowable shear stress (*Fv*) for Doug fir of 95 psi, so we are okay for shear.

Most of the time shear is not a problem, except when we have short, deep beams — that is, when the depth of the beam is greater than about 1/6 its length. Here, the greater distance between the inside and out side fibers causes a greater change in length and thus more shear. Regardless of the beam size, though, never skip the shear calculation.

Deflection Check

Let's move on to deflection. Above and beyond any calculations on the strength of a beam is the idea of "serviceability": Will the beam provide a *useful* horizontal support? "Useful" means that the beam will not only resist breaking, but also will not flex so much that the floor structure is bouncy and unusable, transmitting shocks to other parts of the building. Most beam failures are not breakages, but serviceability failures. Ironically, most beam analyses are only for breakage. This is understandable when you consider that a breaking beam is a threat to life and safety. In residential construction, a breaking beam is extremely rare. The floors that sag and bounce are the ones that cost in repairs and hard feelings.

So we calculate *deflection* — the distance that the beam will sag when the design load is placed upon it. The deflection formula for our "simple beam with a uniformly distributed load" is:

$$D = \frac{(5 \times w \times L^4)}{(384 \times E \times I)}$$

In this formula, E is the *modulus of elasticity*, a property of the material from which the beam is made. *E* is derived by measuring the amount of stress required to stretch a specially sized sample of the material. Here are some typical values:

Material	*E*
S-P-F (constr. grade)	1,300,000 psi
Doug fir (select grade)	1,900,000 psi
Glulam	2,000,000 psi
Aluminum	12,000,000 psi
Steel	29,000,000 psi

The last two figures are included to give some sense of relative magnitude — steel is about 20 times stiffer than wood.

The other item in the deflection formula, *I*, is the *moment of inertia*. Like the *section modulus*, discussed in last month's column, *I* is a property of the beam section. Calculated for a 2x12, where *b* and *h* are the width and depth of the member:

For the quad 2x12, *I* is four times as

$$I = \frac{b \times h^3}{12} = \frac{1.5 \times 11.25^3}{12} = 178 \text{ in.}^4$$

great, or 712 in.^{4}

We almost never calculate the moment of inertia — we just look it up in the tables. Here is the moment of inertia for some common lumber sections:

2x6	20.8 in.^{4}
2x8	47.6
2x10	96.9
2x12	178.0
G3x12	450.0
G5x15	1441.0

Plugging in the values for *E* and *I* we calculate the deflection (*D*) of the Douglas fir girder:

$$D = \frac{5 \times 33.3 \text{ lb./in.} \times (18 \text{ ft.} \times 12 \text{ in./ft.})^4}{384 \times 1{,}900{,}000 \text{ lb./in.}^2 \times 712 \text{ in.}^4}$$

$$= 0.70 \text{ in.}$$

By code, the deflection of the girder must be less than 1/360 of the total length (*L*) of the girder:

$$\frac{L}{360} = \frac{18 \text{ ft.} \times 12 \text{ in./ft.}}{360}$$

$$= 0.60 \text{ in.}$$

Our beam isn't quite stiff enough. The easy out is to add another stick, coming up to a quintuple 2x12. This beam has a moment of inertia of 890 in.^{4}. Recalculating the deflection as above gives 0.56 in., less than the limit of 0.60 in.

But What About Bounce?

So now we're okay for deflection, at least by code. Your friendly building code enforcement officer will accept this structural system. But I'm not so sure that we have done a good design analysis. Most of the code-required analyses just consider the way buildings respond to *static* loadings. These are loadings that are applied and do not move. Real problems occur under *dynamic*, or moving, loads.

The first time I encountered this condition was years ago in a house where the floor system met code for strength and deflection. Yet the floor was so springy that walking across the dining room whipped up a tsunami in the living room fishbowl. The *serviceability* just wasn't there.

To ensure serviceability, I consider the bounciness of the floor. I ask, "Will a 175-pound person falling 6 inches cause the floor to deflect more than 1/2 inch?" I realize that this is an arbitrary measure based on my own experience and intuition. It could certainly use some tuning up, but since I know of no other index, it's a useful working tool. The analysis involves an energy balance calculation, which I'll skip here and just give the result: The girder we're looking at will flex 0.51 inch under this measure — close enough.

However, when you look at the *entire* floor system — the 2x8 joists sitting on the girder — the bounciness is more than an inch. This excessive bounciness suggests that our code-acceptable floor won't really work very well in the real world. Here's what we can do about it: Make the system stiffer with 2x12 joists. If this seems a little excessive, consider that for an extra 200 board feet, you can build a much solider floor system. Eighty to a hundred bucks will make the building a *lot* more comfortable. A little outside the textbook maybe, but think about it. ■

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Curious about the forces that hold a building together — or cause it to fall apart? Address your questions to Practical Engineering, Journal of Light Construction, RR 2, Box 146, Richmond, VT 05477.