FRAMING A Bay Window Roof

by Rick Tyrell

Before I was a custom house builder, I spent 13 years doing remodeling work. It was on a window replacement job that I first encountered a bay window. I knew immediately that the roof framing would be a problem, because it was not a standard hip roof. Somehow I got through the complications of that roof, but I knew that before the next bay window came along, I would have to learn to frame a bay roof the way it should be framed. I had to have a simple, sure-fire method.

There are as many ways to figure roof cuts as there are saw blades for a circular saw, but having a standard procedure saves both time and materials. For the

To frame the irregular hips in a bay window roof, you need to know the rules of right triangles and a little trig

Bay Window Geometry

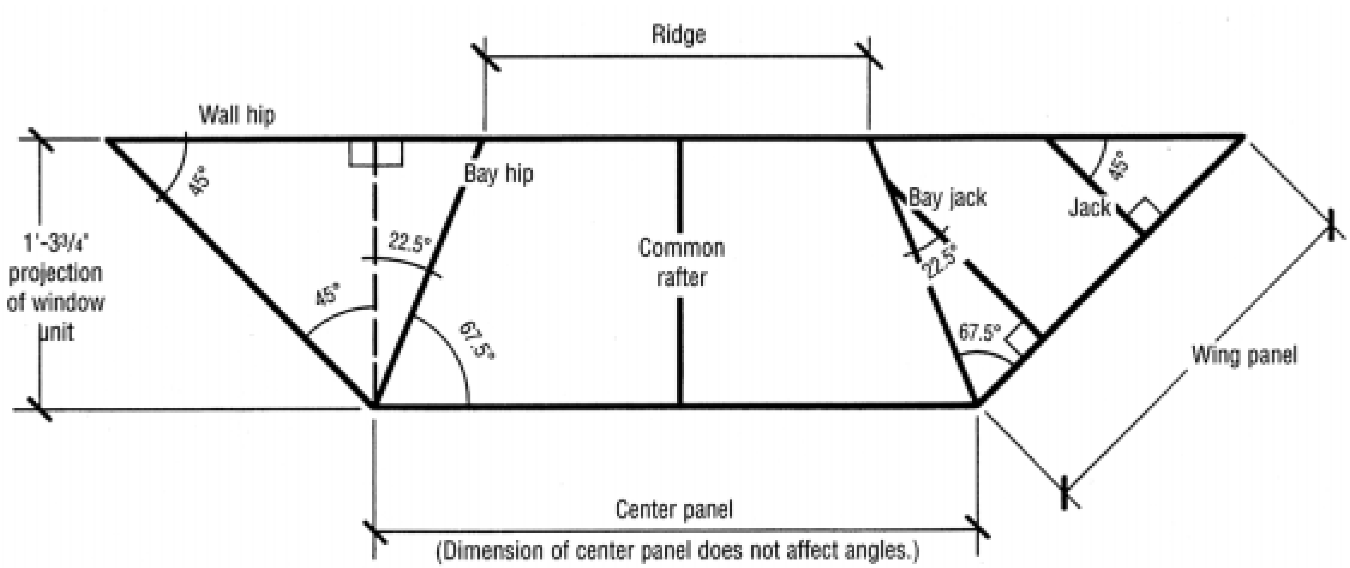


Figure 1. The sum of the angles in any triangle is always 180°. In plan view, you can use this rule — and the fact that the side windows in the bay are at an angle of 45° to the center unit — to calculate the angles formed by the rafters.

methods I use, you'll need a scientific calculator (I use a Texas Instruments TI-35X). I also use a Construction Master III feet-inch calculator to double-check my measurements.

My method consists of a series of formulas that I use each time I have to frame a bay roof. Once I know the angle of the bay and the projection or total run (the distance from the face of the sheathing to the edge of the center panel of the bay), I can determine all the necessary roof cuts. I also use a chart to record all my calculations, so when it's time to cut the rafters all I have to do is refer to the chart.

In this article, I'll present some of the basic rules and formulas of geometry and trigonometry. Those of you who want to know more can consult the two books I used to develop my mathematical method: *Roof Framing* by Marshall Gross (Craftsman Books), which covers all the steps required to lay out both simple and complex roofs; and *Math To Build On: A Book for Those Who Build* by Johnny and Margaret Hamilton (Construction Trades Press), which gives an excellent overview of the math used in the construction trades, including a whole section devoted to calculating right triangles using one side and one angle.

Bay Window Geometry

The bay window I'll be framing in this article is the most common type — a 45° unit. In other words, the three windows of the bay are at 45° angles to each other (see Figure 1). This means that there are two regular hip rafters, which I call the "wall" hips, because they run up the wall against the sheathing; and two irregular or "bay" hips, which run from the peak of the wall hips to the point where the center and side windows intersect. The wall hips are framed at a standard 45°; the bay hips bisect the 135° angle formed by the center and side window units and are framed at 22.5°. The bay window I framed for this article was too small to need jack rafters, but on a larger bay you would also need two types of jacks, one for each type of hip.

Basic dimensions. The bay window in this example has a projection of 1 foot 3 3/4 inches, and the roof pitch is 8/12. You can choose any roof pitch, but I usually match the main roof.

I used 2x4 rafters because structurally, the short run doesn't require anything larger. I won't deal with the dimensions for the cuts at the eaves end of the rafters, because these are typically determined by the width of the fascia and the thickness of the soffit; the angles of these cuts, however, are determined by the formulas I use.

When I install a manufactured bay unit, I adjust the birdsmouth on all the rafters. Because 2x4 rafters are so narrow, I like to leave about two-thirds of the rafter (measured plumb) above the birdsmouth where the rafter crosses the outside of the wall plate. I call this dimension the height-above-plate (HAP), and on this job I used a framing square to set it at $2^3/_4$ inches. The HAP affects the height of the ridge, and it moves the edge of the seat cut towards the center of the plate. The other dimension that affects the ridge height is the double-2x4 plate I lay down at the window unit perimeter to provide solid nailing for the rafters.

Trigonometry

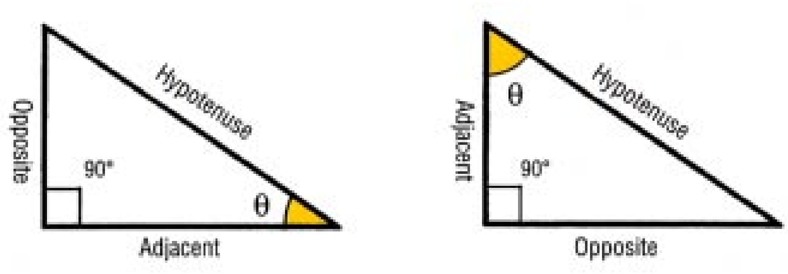
Most carpenters deal with triangles all day long without thinking much about it. In particular, we deal with right triangles — triangles with one 90˚ angle — and most of the math we do on site is used to calculate the hypotenuse using the Pythagorean Theorem ($a^2 + b^2 = c^2$). We use this formula, for example, when checking foundations and decks for square. It's easy because we can usually measure two sides of the triangle, then use the formula to find the diagonal side.

When you can't measure two sides, however, the Pythagorean Theorem doesn't help. Instead, you need to use trigonometry, which is a branch of math that deals with the relationships between the sides of triangles. Two sides of a triangle converge at each of the three angles; when one of these sides is divided by the other, you get a ratio; flip the ratio over and you get a second ratio. Altogether, there are six ratios or functions (Figure 2): *sine*, *cosine*, and *tangent* (abbreviated as *sin*, *cos*, *tan*), and the flipped version of these, *cosecant*, *secant*, and *cotangent* (abbreviated as *csc*, *sec*, *and cot*).

When framing a bay window, which in plan is simply an arrangement of right triangles, we typically know one angle (other than the 90˚ angle) and the length of one leg of the triangle. So the most useful formulas are those that tell us the length of the other sides. After doing a little algebra, we can rearrange the trig formulas to solve for the unknown sides. Which formula we use depends on which dimensions we know.

Trig Functions

(θ = reference angle)



sine $\theta = \frac{opp}{hyp}$ cosecant $\theta = \frac{hyp}{opp}$

cosine $\theta = \frac{adj}{hyp}$ secant $\theta = \frac{hyp}{adj}$

tangent $\theta = \frac{opp}{adj}$ cotangent $\theta = \frac{adj}{opp}$

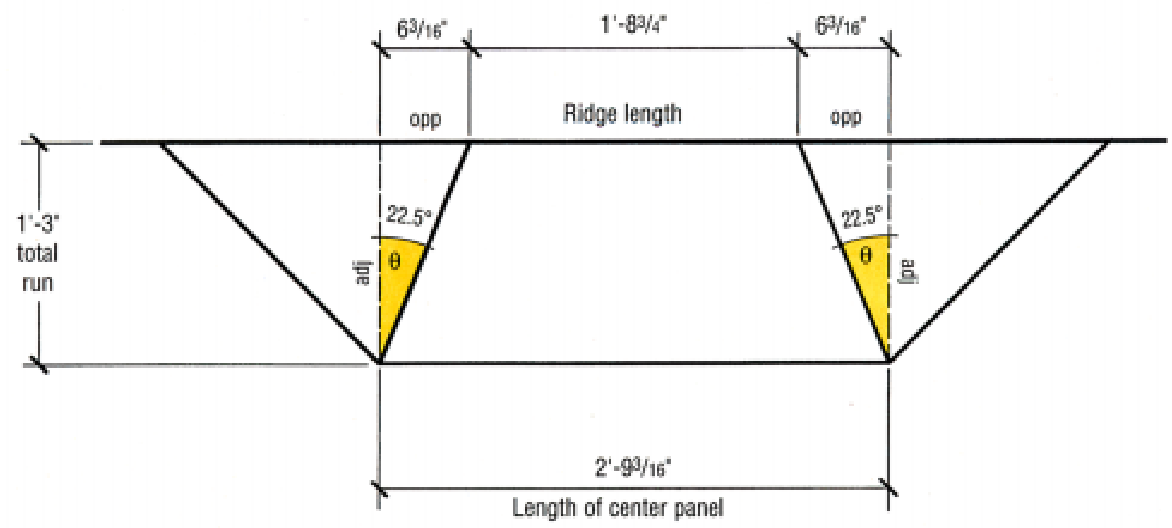
When you know:	To find:	Use this formula:
hypotenuse & 1 angle	opposite	sin θ x hyp
	adjacent	cos θ x hyp
adjacent & 1 angle	opposite	tan θ x adj
	hypotenuse	sec θ x adj
opposite & 1 angle	adjacent	cot θ x opp
	hypotenuse	csc θ x opp

Trig on a calculator: To use trig functions with a scientific calculator, first enter the angle and the trig function, followed by any arithmetic operators and quantities. For example, to find the length of the opposite side when the angle is 22.5˚ and the hypotenuse is 15, strike the following keys in order:

To flip a trig function, use the 1/x key. For example, to enter sec 22.5, key in the following:

Figure 2. The sides of a right triangle are named according to their position relative to one of the non-90˚ reference angles (θ). The six trig functions, which are different combinations of the ratio between two sides, can be used to find the unknown length of a side when one angle and the length of one side is known.

Ridge Length



Known sides and angles:

Center panel: 2'-9 3/16" = 2.765 feet
Total run: 1"-3"* = 1.25 feet
Reference angle: = 22.5°
opp = *tan* θ *x total run*
= 6 3/16" = .5178 feet
*Total run = projection of window unit (1'3 3/4") – 1/2 ridge thickness (3/4")

Ridge length

= *Length of center panel - (opp x 2)*
= 2.765 – (.5178 x 2)
= 1.729 = 1'-8 3/4

Figure 3. In plan view, the bay hips form the hypotenuses of two small right triangles whose reference angles are 22.5°; the adjacent sides are equal to the projection of the window unit (dotted line in illustration), and the opposite sides are short distances along the sheathing. Using the trig formula opp = tan θ x total run, you can find the length of the two opposite sides, then subtract their sum from the length of the center window panel.

For example, when we know the length of the hypotenuse (*hyp*), we can use the formula *opp = sin* θ *x hyp* (the Greek letter Theta — θ — always represents the reference angle); when we know the length of the adjacent (*adj*) side, we can use the formula *opp = tan* θ *x adj*. To find the hypotenuse, we can use the Pythagorean Theorem if we know the length of the two other sides; but we can use trig to save a step and find the hypotenuse directly with the formula *hyp = sec* θ *x adj*. Other relationships are shown in Figure 2, previous page.

3-D trig. If we were laying out a triangular room on the subfloor, we could use these trig formulas to find an unknown angle or the length of an unknown side in one step. But a roof is built in three dimensions, so we're always working with two triangles: one in the horizontal plane and one in the vertical plane (a rafter is really just the hypotenuse of a right triangle with one leg equal to the run and the other to the rise). It's especially useful to think about rafters this way when framing at an odd angle, like the 22.5° angle of the bay hip. To find the unknown length of that hip, we can apply the trig functions to the known dimensions and angles in both the horizontal and vertical triangles the rafter forms. Once we know the unit run of any rafter, we can find its length by multiplying the total run by what Will Holladay referred to as the "line length" (see "Stacking Supported Valleys," 9/97). The line length tells us the dimension in inches along the top of the rafter for every 12 inches of run.

Setting the Ridge

With the bay window unit fastened in place, the first step is to set the ridge, which will lie flat along the sheathing parallel to the center window in the bay unit (Figure 3). The ridge is shorter than the center unit, however, and to find out how much shorter, I use two formulas. The first one gives me the length of the side opposite the 22.5° angle of the bay hip, which I then plug into the second formula to get the ridge length. (I use the Construction Master III feet-inch calculator to double-check.)

To center the ridge on the window unit, I square over a pair of lines, one from each outside corner where the center window meets the wing windows (dotted lines in the illustration). I then measure over $6\frac{3}{16}$ (tan θ x run) from each side and mark the ends of the ridge.

Ridge height. With the ridge position marked side-to-side, I still need to know how high on the wall to fasten it (Figure 4). I start with the thickness of the double 2x4 plates (3 inches), add in the $2\frac{3}{4}$-inch HAP, plus the total rise of an 8/12 common rafter (8/12 x 1.25 x 12 = 10 inches). This puts the height of the ridge at $15\frac{3}{4}$ inches (3 + $2\frac{3}{4}$ + 10). To avoid having to drop the ridge, I bevel half the thickness of the ridge board to match the pitch. In this case, an 8/12 pitch is 33.7° (8÷12, inv tan). I typically rip this bevel on a table saw.

After all of these calculations are completed, I plumb up two lines from the points where I marked the ridge ends, measure up $15\frac{3}{4}$ on each one, and connect the points with a level line. Then I fasten the ridge so its top follows this line.

Common Rafter

The common rafter is simple, but as a preview of the more complex calculations for the hips, I'll figure the length of the rafter using the trig formulas (Figure 5, next page). The line length formula gives me the unit length along the top of the rafter for every 12 inches of common run. Another formula gives me the length of the rafter to the heel cut at the birdsmouth, or what I call the "length to plate"; a third formula gives me the dimension to the plumb cut at the eaves, which I call the "length to fascia." The only difference between these last two formulas is that the length to fascia adds the amount of overhang (in this case, 9 inches) to the total run.

Regular & Irregular Hips

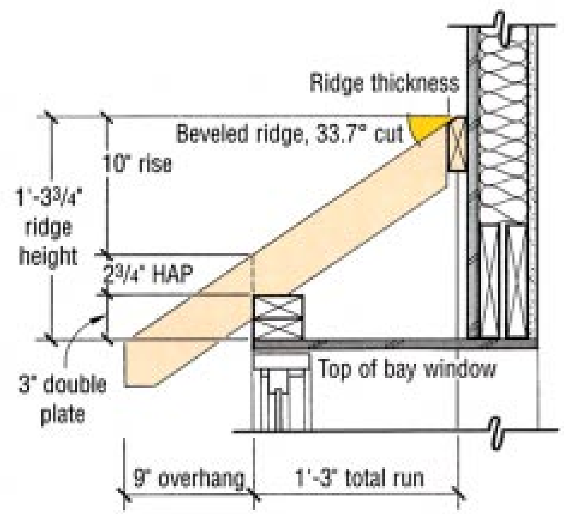
I use the same formulas to find the length to plate and length to fascia for the wall hips. But because the wall hips run at a 45° angle, the unit run is greater than the 12-inch unit run of a common rafter. So to find the line length, I must first find the unit run, which is the hypotenuse of a horizontal right triangle (Figure 6). I can then plug the line length into the two rafter length formulas. I can also use the unit run of the wall hips to set one leg of my framing square to scribe the plumb and seat cuts on the rafters.

Note that the formula for the unit run uses the secant. If there is no "sec" key on your scientific calculator, use a two-key combination — cos 1/x — to find the cosine and flip the ratio.

To find the length of the bay hip, I follow the same steps, but I substitute 22.5° for the working angle (Figure 7). Again, I can use the unit run of the bay hip with the framing square to scribe my plumb cuts.

Top bevels. Both the regular and irregular hips need to be beveled along

Ridge Height



Known dimensions:

Plate thickness (double 2x4 plate) = 3"
HAP (height above plate) = $2\frac{3}{4}$"
($\frac{2}{3}$ of rise across 2x4 as measured using rafter square set at 8/12 pitch)
Rise = pitch x total run x 12
= 8÷12 x 1.25 x 12 = 10"

Ridge height

= Plate thickness + HAP + Rise
= 3 + $2\frac{3}{4}$ + 10 = $15\frac{3}{4}$ = 1'-$3\frac{3}{4}$"

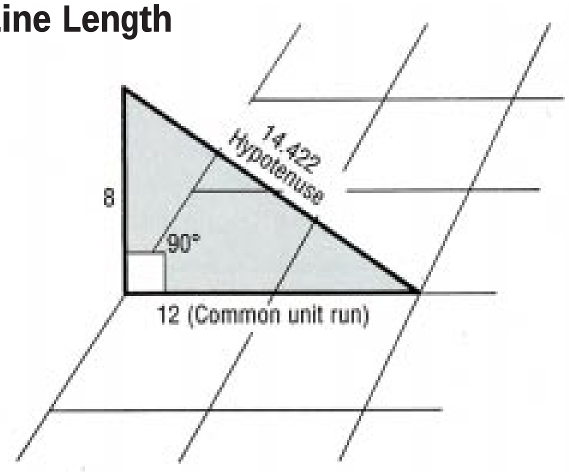
Ridge bevel

= Rise ÷ run, inv tan
= 8÷12, inv tan = 33.7°

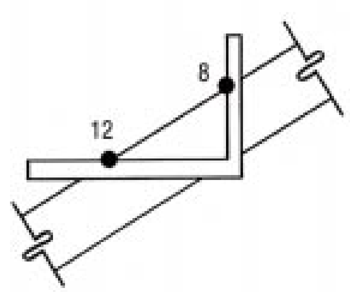
Figure 4. The Ridge Height formula is the sum of the double-plate thickness, the total rise, and the thickness of the rafter (measured plumb) above the seat cut.

Common Rafters

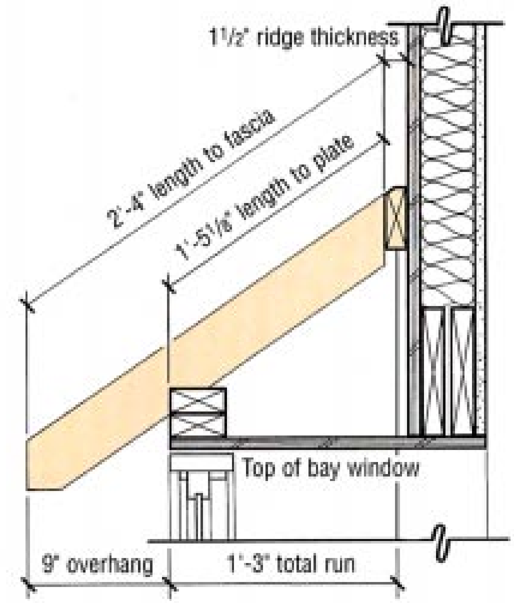
Line Length



Line length = hyp ÷ common unit run
= 14.422 ÷ 12 =1.2018
(hyp = $\sqrt{8^2 + 12^2}$ = 14.422)



Common Rafter Length



Common rafter length to plate

= *(Total run - 1/2 ridge thickness) x line length*
= (1.25 - .0625) x 1.2018
= 1.427 = 1'-5 1/8"
(1/2 ridge thickness = 3/4 inch or .0625 feet)

Common rafter length to fascia

= *(Total run + overhang - 1/2 ridge thickness) x line length*
= (1.25 + .75 - .0625) x 1.2018
= 2.33 = 2'-4"
(overhang = 9 inches or .75 feet)

Figure 5. The top of a common rafter forms the hypotenuse of a triangle whose other two sides are the rise and run of the roof pitch. To find the line length, which is the length along the top of the rafter for every 12 inches of run, divide the hypotenuse by the run. Then multiply the line length times the total run to calculate the overall dimension of the rafter.

the top. This doesn't make much difference on a small bay window, because the hip beams are only 1 1/2 inches wide. But on wider hip beams, the bevel is necessary to keep the sheathing in plane. To cut the bevel, I need to know the width of the rip and what angle to rip it at, and I use a separate formula for each, as shown in Figure 6.

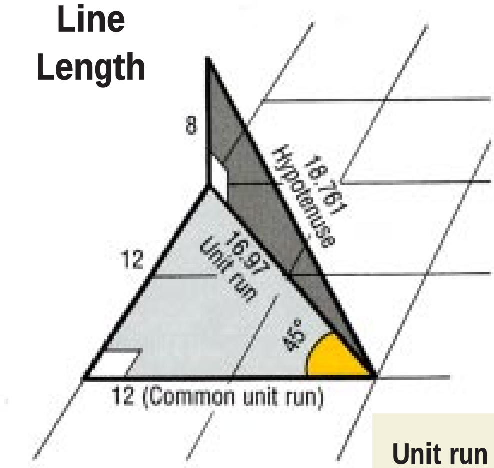
Side cuts. The plumb cut at the peak of the wall hip is cut square to meet the end of the ridge, but at the eaves, you have to set the saw at 45° so the fascia will stay in plane. It's a little trickier, however, to figure these cuts for a bay hip. At the peak, the bay hip meets the ridge at an angle of 22.5° (formulas for the backing, as well as ridge, heel, and tail cuts, are shown in Figure 7).

On a small bay, I use just one common rafter set at the middle of the center unit, but on a larger bay you might need a common rafter at each end of the ridge as well. The bay hip would intersect these common rafters at an angle of 22.5°.

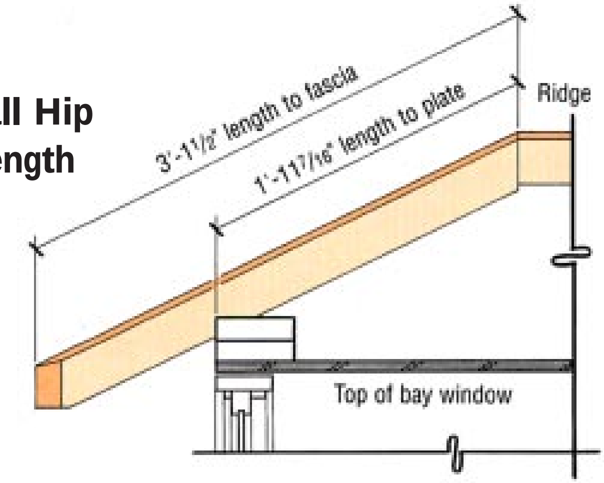
Again, the trig formulas tell me what angle to set the saw at; for the 67.5° angle, you have to scribe the line and make the cut with a hand saw.

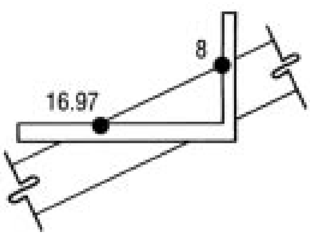
Wall Hip Length

Line Length



Wall Hip Length





Unit run of wall hip

= *sec θ x common unit run*

= 45 cos 1/x x 12

= 1.44 x 12 = 16.97

(sec = cos 1/x)

Line length

= *hyp ÷ common unit run*

=18.761 ÷ 12 = 1.5634

(hyp = $\sqrt{8^2 + 16.97^2}$ = 18.761)

Wall hip length to plate

= *Total run x line length*

= 1.25 x 1.5634

= 1.954 feet =1'-11 7/16"

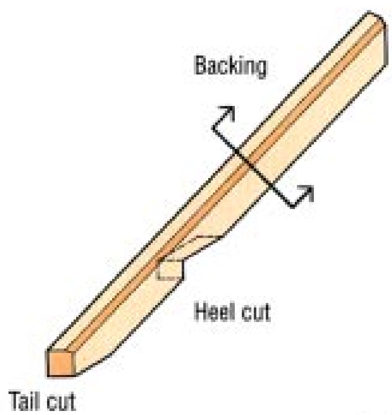
Wall hip length to fascia

= *(Total tun + overhang) x line length*

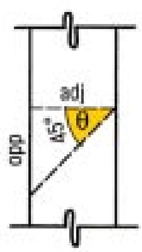
= (1.25 + .75) x 1.5634

= 3.126 = 3'-1 1/2"

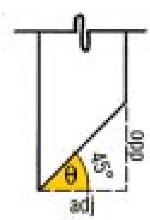
Backing and Side Cuts



Heel Cut



Tail Cut



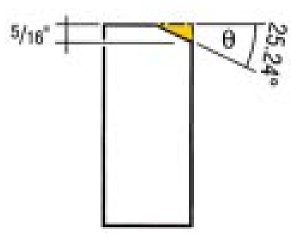
Heel and tail cut

Opp = tan θ x adj

= tan 45 x 1.5

= 1.5"

Backing



Width of rip

= *Rise x 1/2 hip thickness ÷ unit run of wall hip*

= 8 x .75 ÷ 16.97

= .354 = 5/16"

Angle of rip

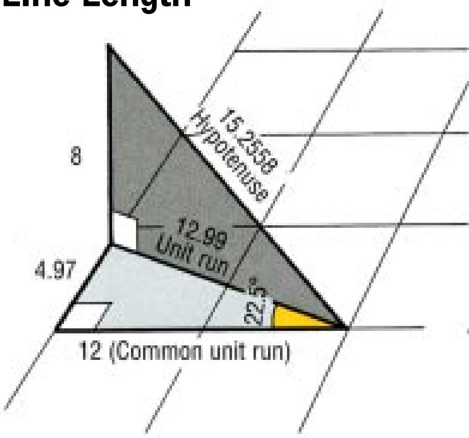
= *(Rise ÷ unit run of wall hip), inv tan*

= (8 ÷16.97), inv tan = 25.24°

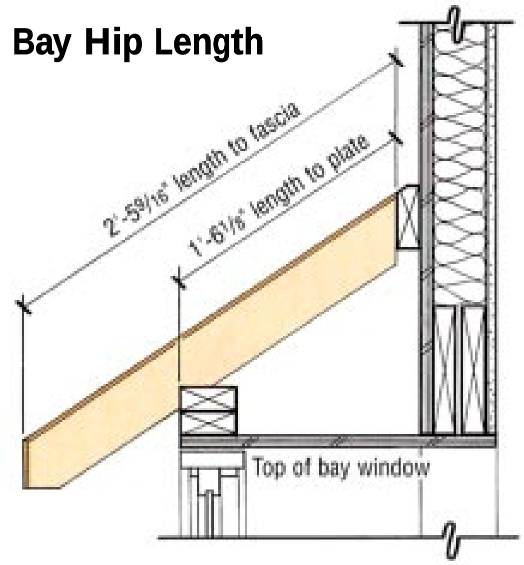
Figure 6. Laid out in the horizontal plane at an angle of 45° to the plate, a standard hip forms the hypotenuse of a right triangle, which in turn is used to calculate the line length (top left). Hip rafters must be backed or beveled to keep the roof sheathing in plane (above). When the rafter tail is left exposed, the side cuts must be cut with the saw set at 45° to provide a tight fit.

Bay Hip

Line Length



Bay Hip Length



Unit run of bay hip

= *sec θ x common unit run*

= 22.5 cos 1/x x 12

= 1.082 x 12 = 12.99

(sec = cos 1/x)

Line length

= 15.2558 ÷ 12

= 1.2713

Bay hip length to plate

= *(Total run – 1/2 ridge thickness) x line length*

= (1.25 - .0625) x 1.2713

= 1.51 = 1'-6 1/8"

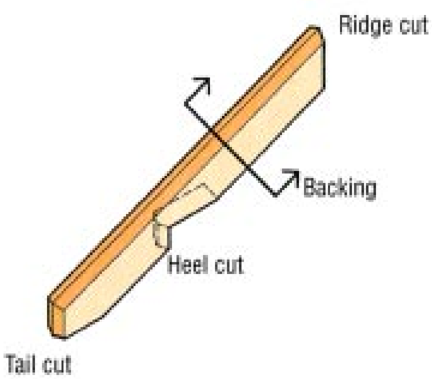
Bay hip length to fascia

= *(Total run + overhang – 1/2 ridge thickness) x line length*

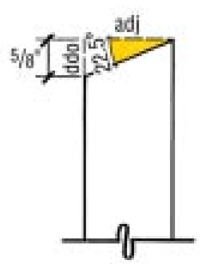
= (1.25 + .75 - .0625) x 1.2713

= 2.463 = 2'-5 9/16"

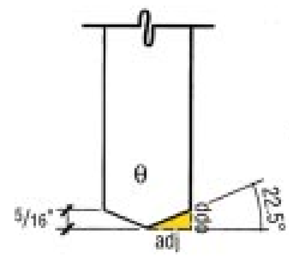
Bay Hip Backing



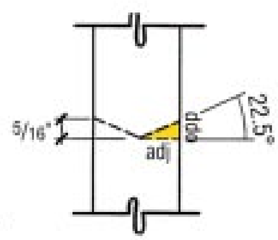
Ridge Cut



Tail Cut



Heel Cut



Ridge cut

opp = tan θ x adj

= tan 22.5 x 1.5"

= 5/8"

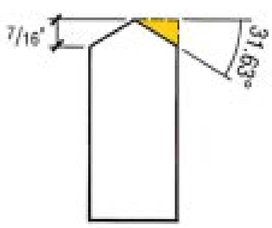
Tail and heel cut

opp = tan θ x adj

= tan 22.5 x .75

= 5/16"

Backing



Width of Rip

= *Rise x 1/2 hip thickness ÷ unit run of bay hip*

= 8 x .75 ÷ 12.99

= .46 = 7/16"

Angle of Rip

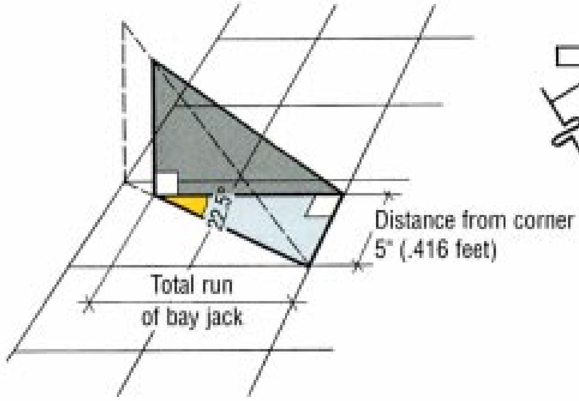
= *(Rise ÷ unit run of bay hip), inv tan*

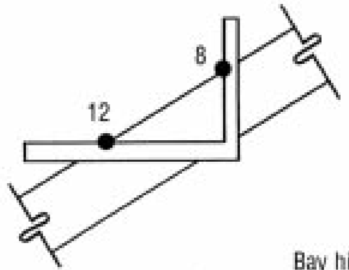
= 8 ÷ 12.99, inv tan = 31.63°

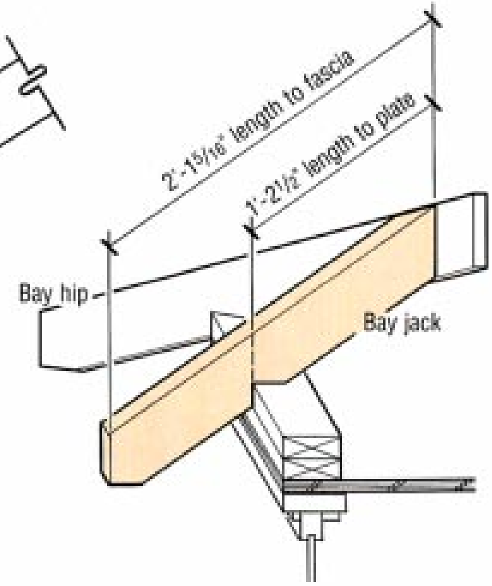
Figure 7. In the horizontal plane, the bay hip is laid out at an angle of 22.5° to the plate. This results in a line length that is longer than that of a common rafter but shorter than that of a standard hip (top left). The bay rafter must also be backed to accept the roof sheathing, and all of the side cuts must be cut with the saw set at a 22.5° angle.

Bay Jack Rafters

Total Run of Bay Jack







Total run of bay jack

= *cot* θ *x distance from corner*

= 22.5 tan 1/x x .416 =1.0043

(cot = tan 1/x) (5 inches = .416 feet)

Bay jack length to plate

= *Total run x line length*

= 1.0043 x 1.2018

= 1.207 = 1'-2 $^{1}/_{2}$"

Total run of bay jack to fascia

= *Total run of bay jack + overhang*

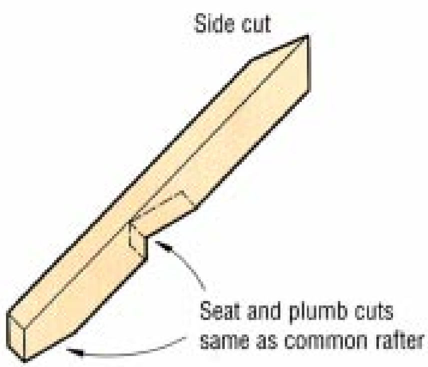
= 1.0043 + .75 = 1.754

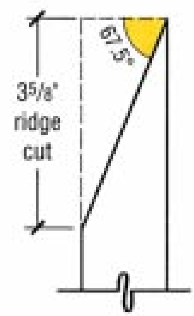
Bay jack length to fascia

= *Run to fascia x line length*

= 1.754 x 1.2018

= 2.108 = 2'-1 $^{5}/_{16}$"





Angle of side cut

= *90° -* θ

= 90° - 22.5° = 67.5°

Length of side cut

Opp. = tan θ x adj.

= tan 67.5° x 1.5"

= 3.62 = 3 $^{5}/_{8}$"

Figure 8. Jack rafters are typically laid out at 16 or 24 inches on-center, but on a small bay window roof, the layout dimensions may vary. Using the "distance from corner" dimension, the formulas shown here will determine the length for jack rafters at any spacing interval. Because a jack rafter is simply a foreshortened common rafter, use the line length for a common rafter at an 8/12 pitch (1.2018).

Jack Rafters

This bay needed only one jack rafter for each wall hip, and was too small to require any jacks at the bay hip. But a larger structure may require a "bay jack," so I've included the formula for calculating both the length and the side cut at the peak. Because most of us are familiar with standard roof framing, we take for granted the "distance from corner" dimension, which is typically the 16- or 24-inch on-center layout. On a small bay window, however, you may want to split the distance between the common and the hip. The formulas I use (Figure 8) will calculate the length of a jack rafter no matter what the layout spacing is. For a bay jack, the angle of the side cut is 67.5°, so you'll have to cut it with a hand saw.

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